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Embodied mathematical practices in (re)designing board games in a linguistically diverse classroom

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ABSTRACT

In this article, we discuss embodied mathematical practices in the context of learners' board game (re)design activities. By focusing on *redesigning* a board game as a pedagogical approach, rather than designing one from scratch, we intended to limit the vast creative design possibilities and engage learners more deeply with the discipline of mathematics. We conducted a design-based research project in a culturally and linguistically diverse Canadian school. Our video analysis identified embodied discourses wherein a student with limited English language proficiency came to be a designer of a board game, while meaningfully engaging in mathematics learning. Our findings demonstrate how the conversations between a newly arrived immigrant student and the teacher in the process of redesigning an existing board game helped the student fully participate in the classroom practice, maximizing the available cultural and linguistic resources.

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Mathematics learning; game design and redesign; board games; English language learners or emergent bilinguals; embodiment

Fully recognizing cultural and linguistic plurality in the classroom calls for expanded notions of *languages* and *participation* in mathematics learning (Barwell et al., 2019; Langer-Osuna et al., 2016; Parks, 2011). Especially for learners with limited language proficiencies in instructional languages, embodied discourse that includes symbiotic use of gestures and language can be a salient part of the semiotic tools for mathematics teaching and learning (Arzarello et al., 2009; Dominguez et al., 2014; Fernandes et al., 2017; Takeuchi, 2018; Takeuchi & Dadkhahfard, 2019; Turner et al., 2013; Zahner & Moschkovich, 2011). Yet in classroom spaces, such non-normative embodied mathematical thinking could be masked (Takeuchi, 2018). In their review, De Araujo et al. (2018) maintained that a learning environment drawing on diverse cultural and linguistic resources, especially for emergent bilingual students, can effectively facilitate students' participation in mathematical discourse. Design of pedagogy with careful attention to what learners bring can change what we consider as languages and participation in mathematics classrooms (Aguirre & Del Rosario Zavala, 2013; Civil & Hunter, 2015; Takeuchi & Esmonde, 2011).

In this article, we discuss learners' board game (re)design activities that could give fuller attention to what learners bring to their learning. Instead of having complete freedom to design games from scratch, we engaged learners in the activities of

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redesigning, i.e. creating new games based on existing board games, through playing, creating new rules, playtesting, and producing games. Our approach aimed to limit the vast creative design possibilities that we observed in previous studies on students' board game designs (Civil, 2002; Kim & Bastani, 2017). While engaging learners in design practices can create opportunities for meaningful disciplinary discourse and identity expression (Kim et al., 2015), learners could be deflected from the discipline of mathematics (Civil, 2002). In redesigning games, learners play a board game selected based on its features, with the shared goal of changing its various elements. Redesign of board games, therefore, sets structured constraints that could enable mathematically rich conversations. This would be accompanied with learners' use of embodied discourse and materials (e.g. dice, boards, pawns) in playing the original game and brainstorming for their new games (Bastani & Kim, 2020). Importantly, game redesign can position learners as individuals who engage creatively with disciplinary ideas (Jaques et al., 2019; Kim & Bastani, 2017; Kim & Gupta, 2017; Kim & Ho, 2018; Kim et al., 2019).

In this specific pedagogical context, we consider how centralizing the emergent bilingual students' diverse communicative resources and drawing on their cultural and experienced understanding could allow them to engage fully in mathematics learning (De Araujo et al., 2018; Fernandes et al., 2017; Moschkovich, 2007; Ng, 2018; Takeuchi & Dadkhahfard, 2019). We conducted design-based research in an inner-city western Canadian school where students are predominantly (approximately 90%) "English language learners" (ELLs)[1] with a diverse range of home languages. We expected that learners would contribute their experience and knowledge through embodied discourse in the context of redesigning a board game with properties relevant to their mathematics learning (i.e. area, multiplication, and estimation). Our analytical focus is embodied mathematical practices surrounding a newly arrived immigrant student with limited English language proficiency in the process of redesigning a board game. Specifically, we attempt to address the following questions: (1) what kinds of mathematics learning opportunities can be observed as a newly arrived immigrant student redesigns existing board games; and (2) how paying attention to an emergent bilingual's embodied discourse could support exhibiting mathematical thinking and drawing on linguistic and cultural resources.

Embodied discourse through game play and (re)design

Mathematical ideas emerge and develop through embodied discourses such as the use of gestures and movements, coordinated with other symbols and tools, in the physical space (Ferrara, 2014; Hall & Nemirovsky, 2012; Kelton & Ma, 2018; Lakoff & Núñez, 2000; Ma, 2017). Hwang et al. (2009) shed light on how learners' heterogenous bodily engagement with objects in communicating mathematical meanings (e.g. through body orientations and pointing and iconic gestures) is central to their conceptual development. Embodied mathematical discourses with tools and materials help not only to characterize what is currently experienced but also to imagine and predict what could happen (Ferrara, 2014; Nemirovsky & Ferrara, 2009). Nemirovsky and Ferrara (2009) demonstrated how students engaged in embodied discourses in imagining and articulating possible cases in an algebra class.

Playing games is one of the basic human activities that are often characterized by mathematics (e.g. actions associated with points, resources associated with cost). Rules are embodied in the game elements. This requires players to imagine possible scenarios, in a similar way to how one engages in embodied mathematical discourses (Nemirovsky & Ferrara, 2009). In the context of computer or video games, embodiment happens through acting or thinking by taking the perspectives of virtual characters (Gee, 2008) or objects-in-motion (Sengupta et al., 2015). In tabletop games (i.e. board games, card games, or other games with physical objects), players' experiences are directly embodied: their movements of game pieces serve as part of the discourse in physical settings. Capitalizing on how embodied discourse enables the imagining of possible actions, researchers showed that board game play or design can support learning of diverse disciplines, such as computational thinking (Berland & Lee, 2011; Kafai & Vasudevan, 2015), and creating and connecting meanings across multiple disciplines (Kim & Bastani, 2017).

Recognizing how mathematics characterizes games, researchers have specifically attended to how playing or designing board games can engage learners in mathematical thinking (Bayeck, 2018; McFeetors & Palfy, 2018; Nasir, 2005; Saxe, 1992). For example, Saxe's (1992) *Treasure Hunt* board game research demonstrated that third and fourth grade learners' mathematical goals shifted and developed within the context of problem-solving required in the game play. He also argued that mathematics is intrinsic to game play, although it is not an end in itself (e.g. working with alternative currency in game economy, estimating the strengths of pawns' positions relative to the opponents to form a strategy). In Nasir's (2005) study on playing the game of dominoes, players of varying ages appropriated game-linked forms (e.g. player's hand, the game board) as a means of game-linked functions (e.g. making a match, scoring). Many tabletop games are embedded in cultural practices (Nasir, 2005) and played across generations (e.g. *Go* in China, *Mancala* in Africa), reflecting everyday mathematics practices. Bringing these cultural practices into the classroom through tabletop games can challenge the dichotomy between academic and everyday mathematics practices (Nasir et al., 2008), as academic mathematics may often seem irrelevant to students and their everyday practices. Building on this body of research and focusing on the case of a newly arrived immigrant student, we examine how, in the process of *redesigning* tabletop games, learners' cultural resources could intermingle with school mathematics practices through an embodied discourse.

Design of the study

We used design-based research (Coburn & Penuel, 2016; Collins et al., 2004) to create learning experiences through board game redesign. During the first phase, the research team explored various design questions informed by the literature and their own teaching and research. In consultation with an expert game designer, we played and redesigned board games and tested them out with our undergraduate and graduate students. The second phase included a series of workshops with seven teachers in a public inner-city school. In these workshops, teachers played games, engaged in redesigning those games, and brainstormed how they could engage their students in similar activities. The teachers with the research team developed game-design learning activities for mathematics tailored to their own classes. In the third phase, students of the teachers in the second phase redesigned board games. In our analysis, we focus on the classroom enactment (the third phase) in one of the classes.

Context and design of the class

The class was a combined grade 3/4 class of Ms. Lennox (pseudonym), which was co-taught with a partner teacher. During January 2018, the teachers had students play a variety of games that had some relevance to their mathematics curricular topics: multiplication, arrays, area models, and estimation. Students and teachers chose *Inversé* as the game to redesign. *Inversé* is a two-person game in which each player has five varying wooden cuboid blocks of the same volume (i.e. 48 cubic units) with different dimensions and colours (e.g. the dimensions of the yellow block are 1 × 6 × 8 grid-lengths and the dimensions of the red block are 2 × 3 × 8 grid-lengths) (see Figure 1). Players take turns to place one block piece at a time on a 12 × 12 grid wooden playing board until one of them can no longer place a piece. The player who can fit the last piece wins the game. There are three simple rules for placing the blocks: (1) same-colour pieces cannot touch; (2) same-colour pieces cannot be placed with the same face as the base; and (3) same-height pieces cannot touch. In Figure 1, the red and black blocks are violating the first rule. In playing *Inversé*, players need to determine which face (e.g. red block in Figure 2: Face A, 2 × 8; Face

Figure 1. *Inversé* being used in the classroom to demonstrate breaking rule #1.

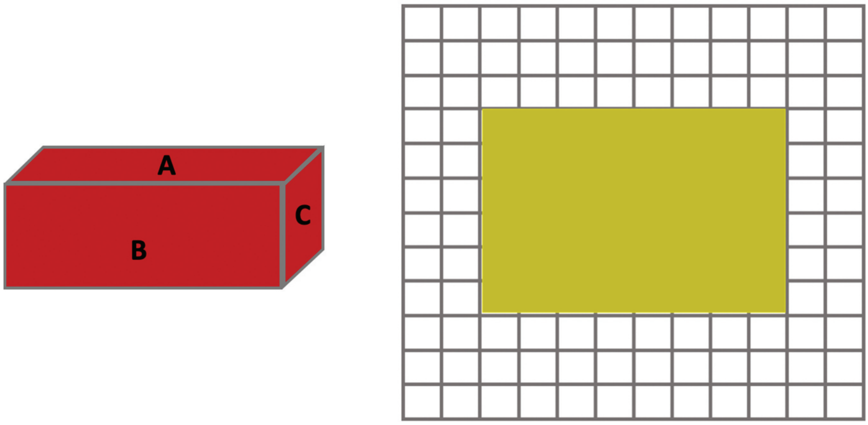


Figure 2. Thinking about areas in playing *Inversé*.

Table 1. The flow of the board game design activities.

Date started[a]	Main design activities
Mid-January	Determining to redesign *Inversé* (from wooden volume blocks to 2D papers), and coming up with individual design ideas
January 31	Forming seven groups to integrate ideas into one game per group
February 5	Drafting their games by incorporating mathematics (area estimation and multiplication) and deciding on the game components (e.g. game board, pieces, dice, etc.) and their sizes
February 7	Play-testing with their peers to make rules clearer and to create rulebooks
February 23	Improving upon their rulebooks by using pictures and texts, and creating titles
February 28	Creating good game copies by using grid papers, modifying board/piece sizes and cutting pieces
March 5	Finalizing game pieces and boards for another class to play and co-created feedback template
March 15	Revising their rules, pieces, and rulebooks based on the feedback

aThis column only indicates the dates that these activities started. The data collection also continued on other dates that are not listed here.

B, 3 × 8; or Face C, 2 × 3) of the rectangular prism would be a good move toward occupying the area of the grid board (i.e. 12 × 12) without breaking the rules. If the opponent places the yellow block with the face of 6 × 8 as the base as the first move (Figure 2), the player might consider using face B of the red block as the base to occupy a larger area, making the next move more difficult for the opponent.

The redesign task was to create two-dimensional (2D) versions of *Inversé* aligned with the local mathematics curriculum for grades 3 and 4. In 2D games, players consider the area of each 2D piece instead of the three different faces of each block. The students went through a series of game redesign activities (see Table 1). Throughout our observations, each class took a typical format of starting as a whole group, i.e. the students sat around Ms. Lennox on the floor to discuss the overall design task for the day. As the students started working in groups, Ms. Lennox joined them to support the development of their ideas and designs. At the end of the class, students gathered back on the floor, shared ideas and progress, and discussed the next plans.

Data collection and analysis

We visited Ms. Lennox's class nine times for sessions of varying length (50 to 90 minutes) over six weeks to collect ethnographic data related to the board game redesign activities. We took field notes, video-recordings of classes, and photos of students' in-progress and final game designs. Ms. Lennox or individual students sometimes wore or held small action cameras for additional recordings of their actions and conversations. We also conducted 30-minute interviews with each group of students and the teachers. In this analysis, we focus on a recent immigrant student's (Jian, pseudonym) series of interactions with Ms. Lennox when she visited his group, and on how their game design evolved. Jian had arrived in Canada from China six weeks before the start of data collection. He worked with two other students for this project. During the group interview, we invited a Mandarin-speaking graduate student to facilitate the interview process.

We identified about six hours of videos of Jian's interactions, a mix of action camera and regular video camera recordings. Through the videos and observation notes, we first identified critical events that showed how Jian's group's game evolved over time. In these evolutions of the game, we also looked for how mathematical concepts, especially multiplication/area models, were represented (i.e. in the process of creating and playing the game). In these identified episodes, we analysed how embodied discourses, drawing on Jian's diverse communicative resources, interacted with symbols and materials used in *Inversé* and his game design, and how Jian's mathematical contributions were intertwined with the culturally relevant play experience that he brought into the game redesign.

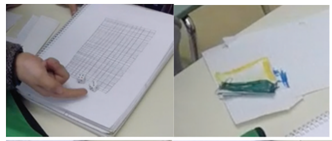
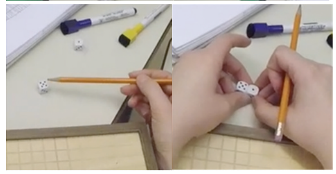
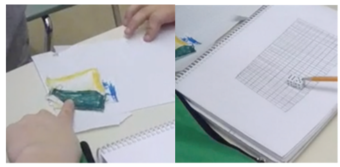
Findings: mathematical and cultural practices of designing a new game – *Blockade*

In this section, we illustrate, through Jian's case, how the context of game redesign created a rich, embodied mathematical and design discourse and how it allowed learners to appropriate their ideas and be positioned as designers. We present six episodes of varying lengths, chronologically organized, to demonstrate how the students' game, eventually named *Blockade*, evolved over time and created connections between Jian's cultural resources and school mathematics. The four main episodes illustrate embodied discourse captured through videos. Through our analyses, we demonstrate the embodied mathematical practices in the redesign of the board game *Inversé* and how these practices enabled Jian to exhibit his cultural resources and become a designer of a board game. Two additional episodes, from which we only had observational data (i.e. notes and photos taken by the researchers), illustrate how the game evolved into its final version.

The initial game rule and its mathematics

Group members had to agree on January 31 on their design directions, especially on how to use rectangles in their games. The groups followed two different methods – players using pre-made rectangular pieces or drawing rectangles with randomly determined sides using dice. Players would need to take turns to fit the rectangles on a grid board. They used the available materials (e.g. grid papers, pencils, colour pens, dice, scissors) to make a rough draft and test their initial ideas. When Ms. Lennox visited Jian's group on

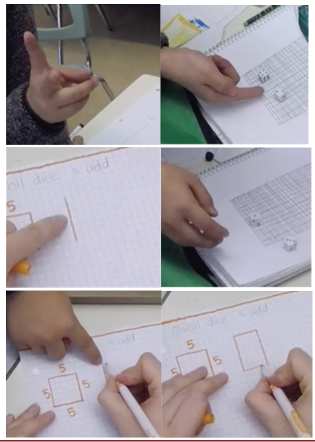
Table 2. Communicating the intended use of dice.

Excerpt	Video captures
(1) Jian: It is one, two, (then) three, or you put these ((rolls two dice and gets 3 and 4)). So, it is (telling) ((pointing to the dice)) you can go . . . seven. ((Ms. Lennox brings a graph paper and *Inversé*, and reminds them of the purpose of the task – making a 2D version))	
(2) Ms. Lennox: ((rolls the dice, gets 1 and 5)) (3) A teammate: Six (4) Ms. Lennox: Are you saying five plus one? ((pointing to the dice one by one)) (5) ((all silent)) (6) Ms. Lennox: So, I am adding these together to get six? ((putting the dice beside each other)) (7) A teammate: Yes. (8) ((Ms. Lennox writes on a grid paper, "1- roll dice and add"))	
(9) Ms. Lennox: So, whatever I roll, I look for a rectangle with the same size? ((pointing to their rectangles)) (10)A teammate: Yes. (11)Ms. Lennox: What if I get five and six ((turns dice to show 5 and 6)) (12)Jian: Eleven (13)Ms. Lennox: What does my rectangle look like? ((pointing to their pieces and dice)).	

January 31, they had some grids drawn on a notebook, two dice, and few coloured rectangle pieces (Table 2, Turn 1 video captures). In the video captures, three rectangle pieces in green, yellow, and blue, which were not fully coloured, are shown stacked together. Jian took the lead in explaining how their game's moves relied on the addition of two rolled numbers. He first pointed to the two dice, which initially showed numbers 1 and 2 (i.e. 1 + 2 = 3), and then rolled new numbers (i.e. 3 + 4 = 7) (Table 2, Turn 1).

Ms. Lennox then brought graph paper and a copy of *Inversé* (After Turn 1, Table 2). She started to draw and write on the paper as they continued the conversation. She demonstrated and stated her understanding in Turns 2 to 8, by rolling the dice, gesturing the addition, and writing the first rule while verbalizing this process (i.e. roll dice and add). In this episode, Jian and Ms. Lennox made explicit connections between the mathematical or gaming terms (i.e. plus, add, and rectangle for mathematics; dice and roll for gaming) and the experienced initial game rule (i.e. roll dice and add). When diverse communicative resources and bodily interactions were integral to the conversation (Turns 1, 2, 4, 6, 9, 11, 13), Ms. Lennox and Jian's group were able to agree that their game involved rolling dice, adding two numbers and picking up a rectangle piece that has one side corresponding to the sum. In connecting with mathematics up to this point, they came up with the idea to pick one of various sizes of rectangles: it required adding numbers, but without using the mathematical concepts they needed to incorporate (i.e. multiplications, arrays or area models). When Ms. Lennox asked, "what does my rectangle look like?" when the number was 11 (Turns 11–13), Jian and other group members did not have an answer. With this initial rule, the possibility of engaging in mathematical conversations emerged, such as how to decide on the variation of rectangles and how many rectangles they might need for a playable game, but instead we observed a change of the rule in the next episode.

Table 3. Drawing a rectangle by rolling two dice twice.

Excerpt	Video captures
(14) Ms. Lennox: What I am asking is, how are you rolling dice and using that to represent a rectangle? What do you think, Jian? What could we do?	
(15) Jian: I think you can throw two ((after showing two fingers to indicate two dice, he rolls them and gets 4 and 2)), because one throw is six ((pointing to the numbers he rolled)).	
(16) Ms. Lennox: Okay, should I draw one side of six? ((drawing a line that spans 6 grids))	
(17) Jian: Yes, yes, six.	
(18) Ms. Lennox: Like this? ((pointing to the line she drew))	
(19) Jian: And one more time ((rolls the dice again, and gets 3 and 1)), and this is four ((pointing to the top of the initial line)).	
(20) Ms. Lennox: Like that? ((drawing one other side))	
(21) Jian: Yes. ((Ms. Lennox drawing two other sides of 6 × 4 rectangle))	

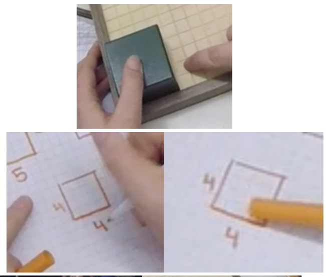
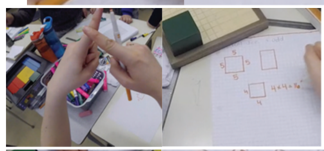
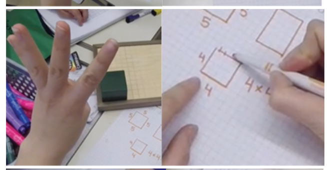
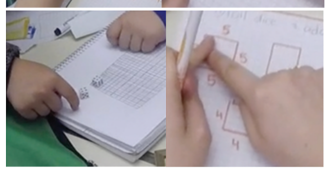
Solidifying the game rules through embodied discourse

In this episode, we present three excerpts that continued from the first episode, where Ms. Lennox facilitated the continuity between the group's game ideas and mathematical models while solidifying their game rules. Before the next excerpt (presented in Table 3), Ms. Lennox led a conversation about the possibility of drawing rectangles with two dice, instead of rolling dice to choose from pre-made rectangles. When Ms. Lennox asked Jian about how to use dice to draw rectangles (Turn 14), he communicated his idea by guiding her through the process.

In Turns 15–19, Jian explained clearly, through his diverse communicative resources, his intent of rolling two dice and adding two numbers twice to create the sides of a rectangle. The answer to Ms. Lennox's question (Turn 14, how are you rolling dice and using that to represent a rectangle?) was enacted through Jian's familiar vocabularies (e.g. throw) and actions (i.e. rolling two dice, pointing to the drawing). After figuring out one way to draw a rectangle, Ms. Lennox went through another possibility of creating a rectangle for the game. This time, she started by discussing multiplication and the area of a rectangle using *Inversé*, shown in the next excerpt (Table 4).

Ms. Lennox reminded them of arrays and multiplication using *Inversé* (Turn 22). When Ms. Lennox asked about the number of squares in the covered area on *Inversé's* board (a 4 × 4 rectangle), Jian readily answered with the multiplied product (16) and enacted the terms (i.e. times, multiply) by drawing the multiplication sign (×) with his finger gestures (Turn 23–26, 33). Jian also tried to differentiate addition from multiplication, adopting the term "plus" as well as drawing the sign (+) on the paper with his finger (Turn 31, 32). It was unclear why Jian interrupted to point out what they were doing was an addition instead of a multiplication. By checking Jian's response (16, Turn 23) through the arrays or the groups of four, however, Ms. Lennox helped the students to create a connection between their game design (i.e. determining how to create a rectangle using two dice) and curriculum expectations. In the next excerpt (Table 5), Ms. Lennox affirmed the changed

Table 4. Gesturing multiplications.

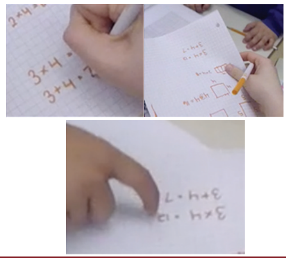
Excerpt	Video captures
(22) Ms. Lennox: We talked about arrays and multiplication. Do you remember that? With these pieces ((showing the *Inversé* pieces)). So, we . . . said the edges fit really neatly inside the grid ((showing the edges of one piece on the grid board)) . . . And we looked at how this is one, two, three, four ((counting the grids on the top side)) and one, two, three, four ((counting the grids on the right side of the block)). So, we have a rectangle that is four and four ((drawing the 4 × 4 square on the grid paper)). . . . In this one, I have four times four. How many squares are there all together ((pointing to the squares inside the rectangle))?	
(23) Jian: Sixteen. (24) Ms. Lennox: How do you know that, Jian? (25) Jian: Four ((drawing × with his right index finger in the air)), four. (26) Ms. Lennox: Like multiply? ((crossing two fingers to create the symbol, ×)) So, four times four ((writing on the paper)).	
(27) Ms. Lennox: Should we double check his answer ((looking at the teammates))? (28) A teammate: Yes (29) Ms. Lennox: Can we count by four's? ((showing 4 fingers)) (30) ((drawing short lines to indicate each set of 4 to guide counting. A teammate counts the squares by 4, "4, 8, . . . "))	
(31) Jian: ((interrupting the flow)) This is plus . . . (32) ((Jian draws + using his finger on the paper, then turns the dice to show 4 and 4)) (33) Jian: Four ((draws × on the paper with his finger)), four, sixteen ((points to what Ms. Lennox has previously written, 4 × 4 = 16)) (34) Ms. Lennox: ((to a teammate)) I think what Jian is suggesting is that instead of adding your numbers together ((pointing to the 5 × 5 square she drew earlier)), you multiply them.	

direction (i.e. creating a rectangle rather than selecting a rectangle piece, based on the rolled dice) and asked students to compare the two ways they had discussed when using two dice (i.e. adding numbers for each side vs. using two numbers for two sides).

In Turns 37–42, Jian supported using "times" for their game by pointing to Ms. Lennox's written expression based on her understanding of their exchange (i.e. $3 \times 4 = 12$, Turn 38), indicating that they might roll two dice once to create a rectangle. Explaining the rationale, he said, "I think it is very quickly" (Turn 40). Ms. Lennox rephrased this utterance and asked for Jian's confirmation, "you can do it faster if you do multiplication?" (Turn 41). This conversation was followed by Ms. Lennox asking students if speed was important for their game. They said "no", and Ms. Lennox left them with the job of coming up with more rules for their game. This excerpt shows how Ms. Lennox guided them in comparing and rationalizing the different choices they had.

This episode with three excerpts indicates how Ms. Lennox engaged with the group to develop the idea of drawing a rectangle, building on the group's initial idea of choosing a piece based on the sum of the two dice's values. This process helped their game become more relevant to the particular models of multiplication for their mathematics learning. Players of their game would potentially engage not just in the addition of two numbers, but in using the area and multiplication models. The exchanges between Jian and Ms. Lennox also demonstrate how Jian could actively

Table 5. Making decisions explicit through comparing the options.

Excerpt	Video captures
(35) Ms. Lennox: In your game, I really like the idea of using dice to make my rectangles. We just need to be clear about how we are using them.	
(36) Ms. Lennox: We talked about three times four . . . ((writing 3 × 4 = 12 and 3 + 4 = 7 on the paper and giving it to the students))	
(37) Ms. Lennox: Which one do you think we should do, plus or times?	
(38) ((Jian points to "3 × 4 = 12"))	
(39) Ms. Lennox: Jian, times? Why?	
(40) Jian: I think it is very quickly.	
(41) Ms. Lennox: You can do it faster if you do multiplication?	
(42) Jian: Yes.	

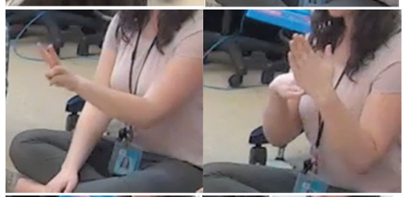
contribute his ideas by embracing embodied communication channels (e.g. visuals, symbolic and physical tools) as an integral part of the conversation. Simultaneously, they created connections between the game rules and school mathematics, as well as between various communicative resources (i.e. gestures, drawings, written expressions, familiar words) and gaming and mathematical terms.

The co-sharing of game design decisions with the whole class

Following the previous episode, the excerpt below shows how Jian led his group's sharing of their work. During the whole class discussion toward the end of each class, Ms. Lennox often asked some groups to share their progress. The exchange reported below between Jian and Ms. Lennox was not just about establishing understanding between them, but more about sharing the ideas with the rest of the class. In the following excerpt in Table 6, Jian started talking about their game while his teammate was hesitating after Ms. Lennox asked his group, "what is the best part of your game?"

The excerpt above shows that the group had determined to use both "plus" and "times" (Turn 37, Table 5) in creating their rectangles, perhaps responding to Ms. Lennox's questioning the importance of speed. It also demonstrates how it was essential for Jian and Ms. Lennox to engage in embodied discourse in order to share the game ideas. In Turn 46, Jian adopted the words "roll" and "dice", and used gestures to demonstrate the play actions for making rectangles. Unlike the conversations they had within the group, Ms. Lennox and Jian could not write, draw, or point to the objects or drawings, but used their gestures. In this context, Ms. Lennox made even more explicit connections between her interpretations and gestures (Turn 48, 50, 53). This episode demonstrates how meaningful communications around game design were socially constructed. It also demonstrates the symbiotic relationship between gestures and languages that Sfard (2009) described: Jian and Ms. Lennox's gestures realized the words (e.g. numbers, shapes) to demonstrate the game ideas, while the spoken words indexed the gestures (e.g. adding it together, big rectangles, Turn 53).

Table 6. Communicating game design decisions between Jian and Ms. Lennox.

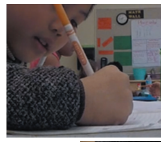
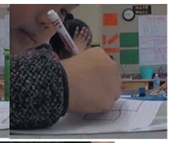
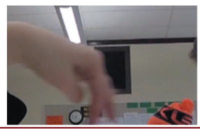
Excerpt	Video Capture
(43) Jian: We, game has a, you're throwing the . . . ((after gesturing of rolling dice, he looks at his teammate)). (44) A teammate: . . . dice? (45) Ms. Lennox: You roll the dice? (46) Jian: Yeah . . . dice . . . Roll four, three ((showing 4 fingers left hand, then 3 fingers right hand)), so seven. And one more time, roll four, two ((4 fingers first then switch to 2 fingers using right hand)), so six.	
(47) Jian: . . . so six and seven, you ((start drawing lines on the flour, from the left side, to the top side)) (48) Ms. Lennox: ((when Jian starts drawing the right side)) A rectangle? ((creating a rectangle using thumbs and index fingers of both hands)) (49) Jian: Yeah.	
(50) Ms. Lennox: So, in your game you roll two dice ((rolling gesture followed by showing 2 fingers to indicate 2 dice)), to make the sides of rectangle? ((using both hands to create right-angled 2 sides)) (51) Jian: Yeah ((nodding)) (52) Ms. Lennox: Okay, then you play the rectangle on the board? ((group members all nodding))	
(53) Ms. Lennox: Interesting. I like how you roll the dice and add it together ((moving her thumb and index fingers close together and then apart twice)), so you get big rectangles ((moving two hands close together and then apart)), not just with (six numbers). That is more challenging.	

The emergence of the game rule, "surrounding"

After the series of episodes described above, Jian proposed a new game rule. He drew this rule from his knowledge and experiences of the Chinese game *Weiqi* or *Go*. Unlike the episodes introduced above, the following excerpt speaks to the process of how a new rule and a new encounter with mathematics emerged. Jian's contribution, based on his creating a connection between *Weiqi* and *Inversé*, played a key role in such emergence.

On February 7, Jian's group sat with another group and Ms. Lennox, because both groups decided to use an identical rule – drawing a rectangle each turn by rolling two dice. Instead of adding two numbers on dice to draw one side, Jian's group decided to go with a simpler rule: drawing two sides of a rectangle with two rolled numbers. Ms. Lennox explained the mathematics of this rule by demonstrating how she could draw a 4×1 rectangle three times to create a 4×3 rectangle when rolling 3 and 4. Another student then tried this method: he rolled 5 and 4 and drew a 5×1 rectangle four times. When Ms. Lennox encouraged the groups to think about how one could decide where to draw rectangles in their games and how to make the game challenging, Jian said, "teacher, I have a game," and grabbed a small whiteboard (see Table 7). This utterance, along with his other utterances (Turn 54, 61, "My game is this."), shows how Jian had a sense of ownership in the game he proposed.

Table 7. Jian's explanation of the rule of surrounding.

Excerpt	Video Capture
(54) Jian: My game is this ((starts drawing grids on a board, Ms. L suggests using a grid paper))	
(55) Jian: You throw ((to Ms. L, pointing to dice))	
(56) Ms. Lennox: So, I roll it?	
(57) Jian: Yes. ((Ms. L gets 1 and 1))	
(58) Ms. Lennox: Can I roll again? ((everyone on the table chuckling))	
(59) Jian: yes	
(60) Ms. Lennox: ((gets 2 and 6)) I roll, and you draw? Do I have to tell you where to draw?	 
(61) Jian: No, my game is this ((starts drawing, 2 × 6, using a red colour marker)) then, I throw.	
(62) Ms. Lennox: Do I take a different colour? ((picks up an orange marker))	
(63) Jian: Yes. ((rolls 4 and 1)) four, one	
(64) Ms. Lennox: Four times one? So, where do I put it?	
(65) ((Jian takes the orange marker from Ms. L and starts drawing))	
(66) Jian: Or, (it) is this . . . ((instead of continuing playing, he draws multiple rectangles)) you are here ((drawing 4 × 1 in orange – he drew three more rectangles in red surrounding the orange one to demonstrate))	
(67) Jian: I am winner ((showing Ms. L what he drew, pointing to one of the red rectangles)).	
(68) Ms. Lennox: So, I lose? So, the goal is to surround your opponent? ((circling her hand in the air above Jian's drawing to indicate the orange rectangle is surrounded by red ones))	
(69) Jian: ((nods))	

In the excerpt above, Ms. Lennox asked to roll again (Turn 58) when she rolled the smallest number combination (1 and 1). Ms. Lennox and Jian agreed with chuckles that rolling small numbers may not be desirable regardless of Jian's new game idea, and Ms. Lennox rolled dice again (Turns 57–59). Jian explained the new rule of surrounding the opponent's rectangle by using two markers with different colours (Turns 60–67). Ms. Lennox checked her understanding of Jian's idea by saying and gesturing what she understood: "The goal is to surround your opponent" (Turn 68). Ms. Lennox suggested that the group play with this new rule and check on the rules.

On March 1, one researcher (Kim) saw Jian's group playing their game and asked Jian about his strategy. He answered "weiqi" and then repeated "wei, wei." He also wrote a Chinese character on her notebook (see Figure 3). As he could not find a proper English word, he demonstrated it by surrounding a classmate with another student and the researcher. When the researcher said, "we are surrounding him," he seemed happy to hear the word and repeated "surround, surround," which Ms. Lennox had also used some time before (Table 7, Turn 68).

This set of events demonstrates Jian's participation in mathematics learning, which became possible through the approach of designing games and his positioning of embodied discourse in communicating his ideas. On March 21, a Mandarin-speaking graduate student helped with interviewing Jian. Jian explained that his idea came from the Chinese game *Go*, which is called *Weiqi* (围棋) in China. Similar to *Weiqi*, each player in Jian's game decides where to place a rectangle for the purpose of surrounding the opponents' rectangle(s).

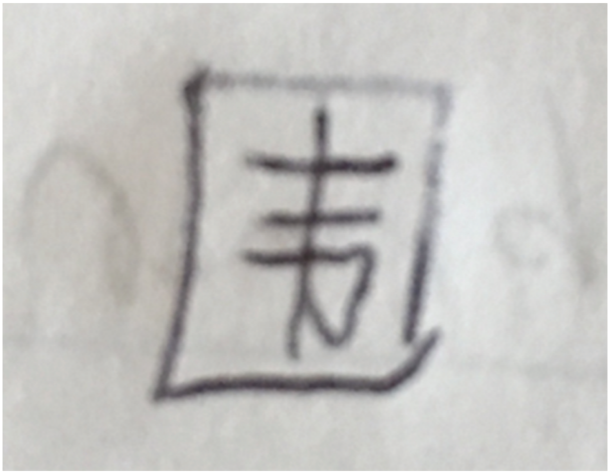


Figure 3. Jian's writing of the word Wei.

Exploring different rules with dice

Jian's group started playing this new game of surrounding opponents and adding more rules, after the episode described in Table 7. We do not have video-based data of Jian exploring these rules, but will discuss them to demonstrate the evolving game designs. In their game, players took turns to draw rectangles with the goal of surrounding the opponents' rectangle(s). Players used different colours to distinguish their rectangles (as Jian demonstrated in Table 7). Unlike *Inversé*, the same colour and the same length sides could touch each other. They also decided to write down the calculated area with the same colour (see Figure 4). This rule helped to mark which rectangles belong to which player for the game, while seamlessly connecting with the school mathematics element of using the area-multiplication model.

On February 9, other students played Jian's game, but with two dice that had numbers 1 to 10 instead of 1 to 6 (see Figure 5, dice in green). Ms. Lennox asked, *"why did you decide to have dice with bigger numbers?"* One of Jian's groupmates explained that bigger numbers could cover bigger space and the play would go faster.

It seemed that the dice with bigger numbers helped them cover their grid paper faster, but without surrounding any opponent's rectangle. The conversation with Ms. Lennox led to a discussion about the winning rules and the possibility of having a tie. Jian suggested that a winner could be decided by the sum of the calculated areas. Ms. Lennox challenged Jian saying that such a rule would not be based on players' strategy but more on their luck. Playtesting and conversations with Ms. Lennox and classmates helped the group reify their rules and continue to experiment with how they could use dice to make their game more strategy-based. On March 1, we noticed that they had changed their dice back to those with 1 to 6, and were making their board with 30 × 30 grid after multiple playtests. During our interview, in relation to using dice, Jian suggested that their game, finally named *Blockade*, could have three versions with different levels of difficulty, based on his experience with *Go*: the easiest version using regular six-number dice (1 to 6), the middle level using higher three-number (3 to 5) dice, and the most difficult one using lower three-number (1 to 3) dice.

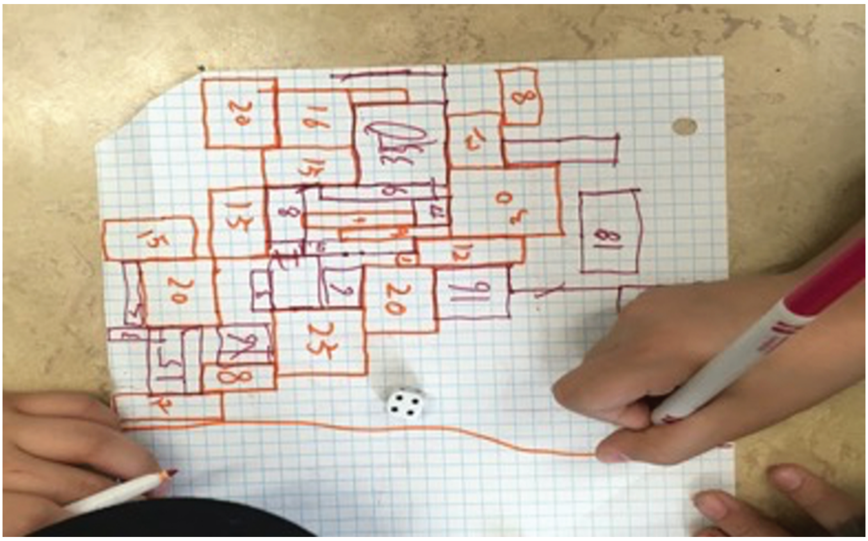


Figure 4. Jian’s group playtesting their game.

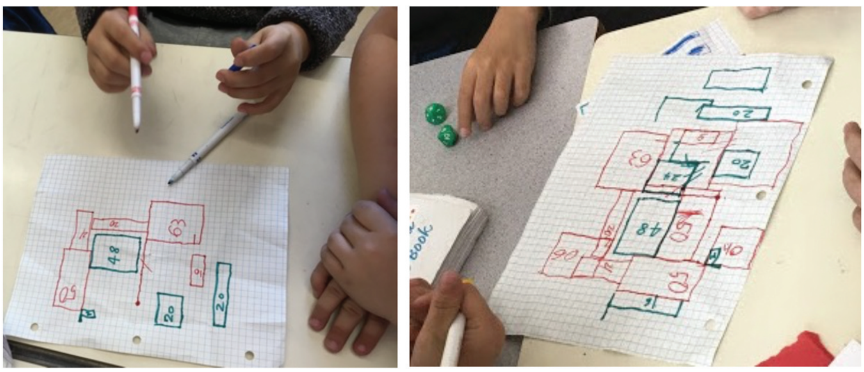


Figure 5. Testing the game with dice with more numbers (1–10).

Sharing Blockade with others

In their class on February 28, Jian’s group glued nine pieces of 10 × 10 grid squares on cardboard to create a 30 × 30 grid board. Ms. Lennox laminated the board on March 5, and they played their game multiple times to test it, and to create their rule book (Figure 6a, the hand-written rulebook shown on the right side of the board). The students’ games were played during the school’s game night on April 10 (Figure 6b), for which they prepared a typed-up rulebook with photos of their game with the help of Ms. Lennox (Figure 6c). We included Figure 6 to demonstrate the completed game to be played by others. The school library now has a collection of student-created games that teachers and students can borrow and play.

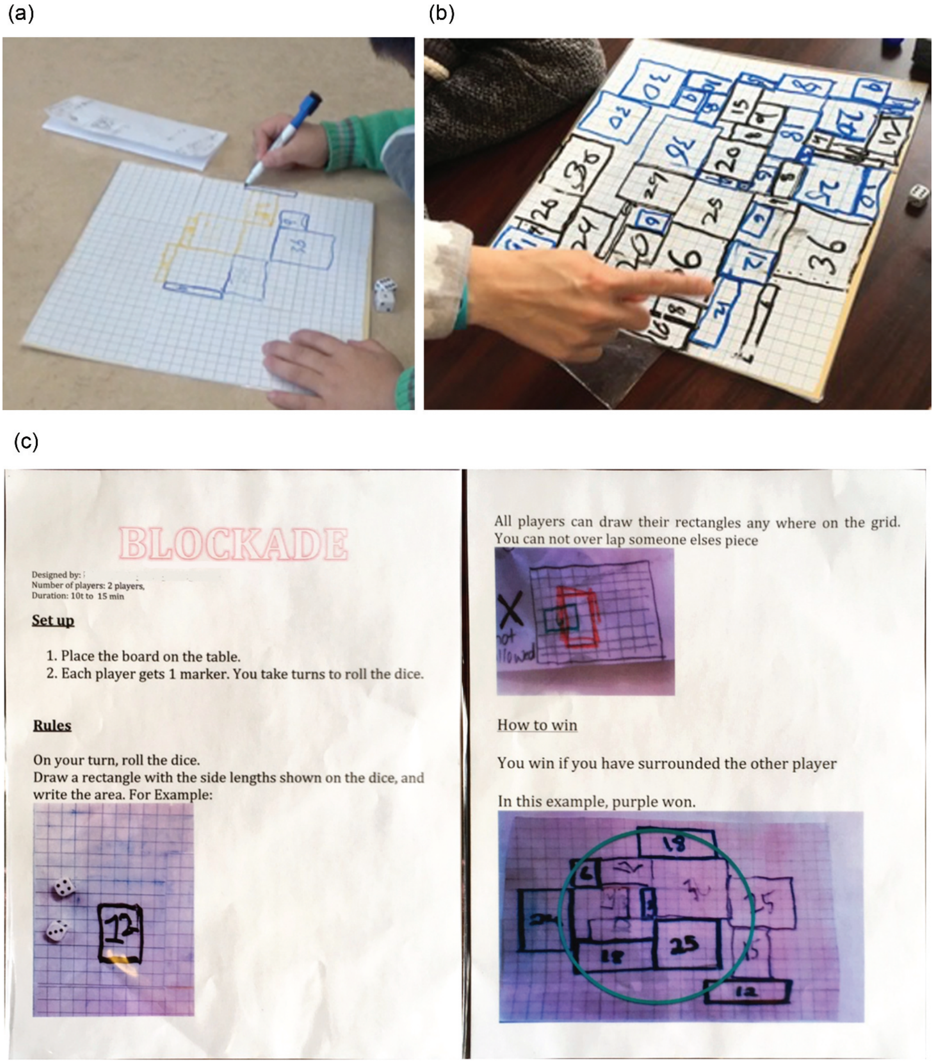


Figure 6. Completed game, *Blockade*: (a) playtesting with their laminated board with their draft rule book on the side (March 5); (b) Jian playing with another classmate on a school game night (April 10); (c) final version of their rulebook.

The final version of *Blockade* shows how the redesigned game and its play could blur the boundaries between school mathematics and game play. Students had made the area-multiplication model relevant in this game context. *Blockade* resembled and deviated from both *Inversé* and *Weiqi*, combining the rules of both games as well as adding new rules. The winning rule of *Blockade* was closer to *Inversé*, where creating a condition (i.e. the opponent can no longer fit a block in *Inversé*; a player surrounds at least one of the opponent's rectangles in *Blockade*) determines a winner. *Blockade*'s rule of surrounding the opponent was drawn from *Weiqi*, but its point system was not adopted (see Davies, 1992 for more information on the rules of *Weiqi*). The students' redesign activities created a connection between school mathematics, the school-introduced game, *Inversé*, and Jian's culturally relevant game, *Weiqi*.

Discussion

In this article, we highlighted how the pedagogy of board game redesign amplified embodied mathematical practices and enabled a newly arrived immigrant student's cultural resources to relate to school mathematics. The activities of redesigning – i.e. playing an existing game, creating new rules, playtesting new games, and producing them – provided the structured constraints for learners to engage in mathematically rich conversations, using embodied discourse (Bastani & Kim, 2020).

Our findings demonstrated the mathematical opportunities that Jian experienced through redesigning *Inversé* using embodied discourse. By communicating mathematical meanings through embodied discourse, Jian developed both mathematically sound and balanced rules and imagine possible scenarios with new rules (Nemirovsky & Ferrara, 2009). In earlier episodes, Jian's group and Ms. Lennox explored the initial game rule: how to determine what their playing piece would look like in each turn. In this process, they explored using mathematics, as seen in Figure 7, to identify (a) or to create (b, c) rectangles using dice. Through the conversation, adding the numbers of two dice (Figure 7a) was recognized as incomplete (i.e. indicating only one side), with group members having difficulty imagining what their rectangles would look like (see Table 2). They discussed how using only multiplication (Figure 7b) would make the game go faster (see Table 5) whereas doing the addition twice (Figure 7c) would make the game more challenging (see Table 6). Ms. Lennox sometimes deviated from discussion of the game rules to engage some group members with the relevant mathematics concepts (see Table 4). However, the area models were still explored in the context of the game design, and more specifically how their design choices would influence the game play (e.g. how and where to draw a rectangle in order to progress strategically

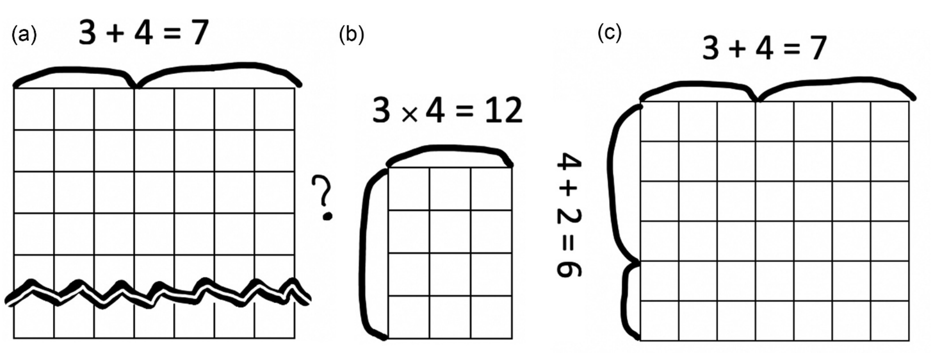
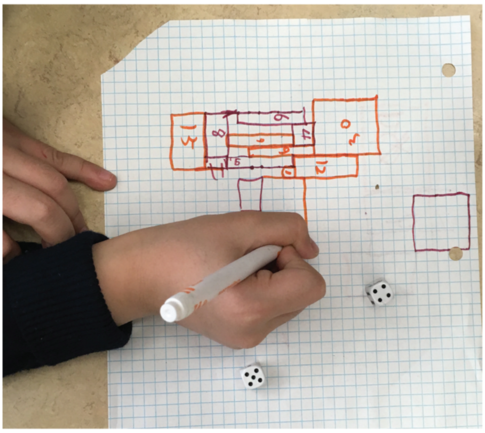
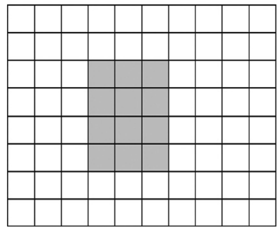
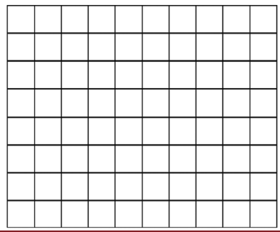


Figure 7. Earlier uses of mathematics in exploring the game rules (the first three episodes): (a) choosing a piece based on the addition of two numbers; (b) drawing a rectangle based on two numbers rolled; (c) drawing a rectangle by rolling two dice twice.

Table 8. Dynamic use of mathematics in playing *Blackade* in comparison to typical worksheet problems.

Blockade	Typical Worksheet Problem
On your turn, roll two dice. *Estimate the area of a rectangle to draw with the two rolled numbers.* *Determine where to place the rectangle in order to surround the opponent.* *Draw the rectangle and write the area.* 	*Write a multiplication sentence* ___ × ___ = ___  *Draw an area model* 3 × 4 = ___ 

toward winning). Their design conversations and the expected play based on the rules did not exclusively belong to mathematics or game design, but had the aspects of both, expanding their attention to the mathematical ideas.

Our findings, therefore, illustrate that the process of redesigning *Inversé* was the process of co-creating cultural and mathematical practices between Ms. Lennox and the students. It is also the process of blurring the boundaries between disciplinary and cultural knowledge (Nasir et al., 2008). Jian's group's final game, *Blockade*, which adopted Jian's cultural knowledge of the game of *Weiqi*, embodied a dynamic use of an area model in comparison to typical area model practices (see Table 8). In playing *Blockade*, players need to compare the area based on their rolled numbers with the empty spaces on the grid board. At the same time, they need to place their rectangles by anticipating the opponent's moves, in order to make a better progression in surrounding one or more of the opponent's rectangles. The designers and players of this game, therefore, engaged with the concept of area from various viewpoints and in meaningful ways, i.e. in creating and participating in the rule-based system of a game. By contrast, typical worksheet problems of area and multiplication generally ask students to merely write a multiplication sentence based on a drawn area model or to draw an area model based on a multiplication sentence. What was shown in Jian's case was a more dynamic use of an area model for multiplication in the context of game play and design.

Jian's contributions demonstrated how his cultural, linguistic, and mathematical resources became meaningful in this context. Ms. Lennox helped to make the experience of playing *Inversé* relevant to school mathematics and to the game redesign process for all students. The connection among the experiences of playing cultural games (*Weiqi* for Jian), redesigning games, and doing school mathematics was co-created by the students and Ms. Lennox. In Jian's case, he showed that he was comfortable with multiplications and area models. He was able to explore how the different uses of mathematics in games could change the game rules and play experience, such as different uses of dice (e.g. rolling two dice twice, rolling dice with higher or lower number sets) for *Blockade*. In this process, both Ms. Lennox and Jian engaged in embodied discourse as an integral part of mathematical and game design conversations. By attending to embodied mathematical practices, competences and resources embedded in informal cultural practices can come into contact with school mathematics practices (Civil, 2007, 2016; González et al., 2001; Takeuchi, 2018), especially for students from non-dominant backgrounds.

Conclusions

Our findings add to the discussion of embodied mathematical practices in linguistically and culturally diverse classrooms by capitalizing on the pedagogical approach of game redesign. Playing and choosing a game together first created a common ground for all students, while redesigning a game opened up for students' diverse gameplay experiences to enter the classroom. Our finding suggests that the redesign process also allowed for the embodied discourse to emerge. It is important to note that the original game *Inversé* and its play did not strongly privilege the dominant language (i.e. English) proficiency, and thus allowed students with non-dominant backgrounds to participate fully. We suggest that teachers could make conscious choices of games, together with students, not only to address the curricular expectations, but also to support all students' participation through embodied mathematical discourse. Our findings showed how the choice of the game combined with teacher's leveraging of students' emergent design goals helped them engage more deeply in using mathematics. specifically, this can become possible through the teacher's efforts to achieve shared understandings with the students about their ideas and practices through embodied mathematical discourse. We suggest that the pedagogical approach of redesigning tabletop games has a great potential to synthesize embodied mathematical discourse, learners' cultural and linguistic resources, and school mathematics learning. Such a synthesis can open up participation in mathematics discussion to a wider range of learners.

Note

1. We are aware of the problematic nature of the deficit framing of the label "English language learners" as discussed by Gutiérrez and Orellana (2006) and the risks of perpetuating colonial representation and othering as discussed in Takeuchi (2021). We use this term here, with quotation marks, to point out that the term is still used in the school context.

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